University of Sask

Department of Mathemati

Math 225 Spring 2003

May 30, 2003 — Test #1 — 2 hours

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The examination consists of two parts, Part A and Part B, each worth 20 points. The points however, will be used in a formula to calculate your test grade.

- *Encode* your student number correctly on your opscan sheet.
- *Print* your name and student number on your opscan sheet.
- Answer all questions of Part A *in pencil* on your opscan sheet. There is no penalty for a wrong answer in Part A.
- Answer all questions in Part B in the answer book provided.
- One formula sheet is permitted. No calculators.

PART A

Fill in the bubbles on your opscan sheet corresponding to the correct answers. Each problem in this section is worth 1 point.

Question 1. The value of $2\vec{a} - 3\vec{b}$ where $\vec{a} = (1, -2, 3)$ and $\vec{b} = (1, 1, -1)$ is

(A) $(0, -2, 1)$ (B) $(-1, 3, 0)$ (C) $(-1, -7, 9)$ (D) $(1, -1, 2)$ (E) $(1, -2, 3)$
(F) $(2, -4, 6)$ (G) $(0, 0, 0)$ (H) $(3, 3, -3)$

Question 2. The value of $\vec{a} \cdot \vec{b}$ where $\vec{a} = (1, 0, 3)$ and $\vec{b} = (3, 1, 2)$ is

(A) 3 (B) 4 (C) 0 (D) $\sqrt{10}$ (E) 10
(F) 9 (G) -3 (H) 6

Question 3. The length of the vector $(-1, 3, 1, 2)$ is

(A) $\sqrt{10}$ (B) 3 (C) 15 (D) 4 (E) 225
(F) 0 (G) $\sqrt{15}$ (H) $1-$

Question 4. The value of $\vec{a} \times \vec{b}$ where $\vec{a} = (1, -2, 1)$ and $\vec{b} = (-1, 1, 1)$ is

no right ans.

(A) $(-3, -2, 3)$ (B) -3 (C) $(1, -1, 0)$ (D) $(1, 1, 0)$ (E) $\sqrt{3}$
(F) $(2, -3, 0)$ (G) $(-3, 0, 0)$ (H) $(0, 1, 0)$

Question 5. The distance between the two points $(-2, 1, 2)$ and $(1, 4, 0)$ is

(A) 484 (B) 4 (C) 6 (D) 2 (E) $\sqrt{22}$
(F) 16 (G) $\sqrt{8}$ (H) 22

Question 6. The value of t such that $(t, 1, 2t)$ and $(1, 1, -2)$ are orthogonal vectors is

(A) $\frac{1}{3}$ (B) 1 (C) 5 (D) 0 (E) 3
(F) $\frac{1}{5}$ (G) $\frac{2}{3}$ (H) -1

Question 7. The line which contains the points $(1,0,2)$ and $(3,0,3)$ meets the plane $z=0$ at the point

(A) $(-3,-2,3)$ (B) -3 (C) $(1,-1,0)$ (D) $(1,1,0)$ (E) 0
(F) $(2,-3,0)$ (G) $(-3,0,0)$ (H) $(0,1,0)$

Question 8. The equation of the plane containing the points $(1,0,2)$, $(2,2,1)$, and $(-1,1,0)$ is

(A) $3x-4y-5z+7=0$ (B) $3x-4y-5z+1=0$ (C) $3x-y-5z+1=0$
(D) $3x+y-5z-7=0$ (E) $-3x-4y-5z+7=0$ (F) $3x-4y-5z=0$
(G) $3x-4y-2z+2=0$ (H) $x-y-z+1=0$

Question 9. The component of $\vec{b}=(3,-2,2)$ on $\vec{a}=(2,0,1)$, or $\text{comp}_{\vec{a}}\vec{b}$, is

(A) 6 (B) $\frac{8}{\sqrt{17}}$ (C) $\frac{8}{5}$ (D) 8 (E) 0
(F) $\frac{8}{17}$ (G) $\frac{8}{\sqrt{5}}$ (H) 1

Question 10. The projection of $\vec{b}=(3,-2,2)$ on $\vec{a}=(1,0,1)$, or $\text{proj}_{\vec{a}}\vec{b}$, is

(A) $(1,0,1)$ (B) $(\frac{15}{17},-\frac{10}{17},\frac{10}{17})$ (C) $\frac{\sqrt{5}}{\sqrt{2}}$ (D) $(\frac{5}{2},0,\frac{5}{2})$ (E) $\frac{5}{\sqrt{2}}$
(F) 0 (G) $(3,-2,2)$ (H) $(\frac{15}{17},\frac{10}{17},\frac{10}{17})$

Question 11. The tangent line to the curve $\vec{r}(t)=(1+t,\sin t,t^2-2)$ at $t=0$ meets the plane $x=0$ at

(A) $(0,2,-2)$ (B) $(0,1,1)$ (C) $(0,-2,-1)$ (D) $(0,-1,-2)$ (E) $(0,1,3)$
(F) $(0,3,1)$ (G) $(0,0,0)$ (H) $(1,0,1)$

Question 12. If $\vec{u}(t)$ and $\vec{v}(t)$ are two curves such that

$$u(0)=(1,-2,1),\quad v(0)=(0,1,1),\quad u'(0)=(1,-2,0),\quad v'(0)=(-2,1,0)$$

then the value of $(u\cdot v)'(0)$ is

(A) 4 (B) -4 (C) 1 (D) 0 (E) 3
(F) -6 (G) -1 (H) -3

Question 13. With exactly the same information as in the previous question, the value of $(u\times v)'(0)$ is

(A) $(0,0,4)$ (B) $(1,3,-4)$ (C) $(4,1,-2)$ (D) $(1,1,1)$ (E) $(0,0,-4)$
(F) $(0,0,0)$ (G) $(-3,-3,-2)$ (H) $(1,0,0)$

Question 14. If $\vec{r}(t)=(t,1,t^2)$ then $\displaystyle\int_0^1 \vec{r}(t)\,dt$ is

(A) $(1,-1,1)$ (B) $(1,1,1)$ (C) $(0,0,1)$ (D) $(0,0,0)$ (E) $\frac{11}{6}$
(F) $(\frac{1}{2},1,\frac{1}{3})$ (G) $(1,0,2)$ (H) $(1,1,0)$

Question 15. The velocity of a particle moving by the curve $\vec{r}(t)=(\sin t,\cos t,t^2)$ at $t=\frac{\pi}{4}$ is

(A) $(1,0,0)$ (B) $(0,0,0)$ (C) $(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},\frac{\pi}{2})$ (D) $(\sqrt{2},\sqrt{2},1)$ (E) $(0,\sqrt{2},\frac{\pi}{4})$
(F) $(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},\frac{\pi}{4})$ (G) $(0,0\frac{1}{3})$ (H) $(\sqrt{2},\sqrt{2},\frac{\pi}{4})$

Question 16. The unit tangent vector to the helix $\vec{r}(t)=(\cos t,\sin t,-t)$ at $t=\frac{\pi}{4}$ is

(A) $(-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0)$ (B) $(-\frac{1}{2},\frac{1}{2},-\frac{1}{\sqrt{2}})$ (C) $(\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}})$ (D) $(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}})$ (E) $(1,0,0)$
(F) $(\frac{1}{2},\frac{1}{\sqrt{2}}),\frac{1}{2})$ (G) $(0,-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}})$ (H) $(0,1,0)$

Question 17. If $z = x^2y + 3xy$ then $\dfrac{\partial z}{\partial x}$ is

(A) 0 (B) $x^2 + 3x$ (C) $2xy$ (D) $2xy + 3$ (E) $3xy$
(F) $x^2y + 3y$ (G) $2xy + 3y$ (H) $2x + 3$

Question 18. If $z = \sin(x^2 + xy)$ then $\dfrac{\partial z}{\partial y}$ is

(A) $y\cos(x^2 + xy)$ (B) $\cos(x^2 + xy)$ (C) $(2x + y)\cos(x^2 + xy)$ (D) $3x\cos(x^2 + xy)$
(E) $\sin(x^2 + xy)$ (F) $x\cos(x^2 + xy)$ (G) $2x + y$ (H) $y\cos(x^2 + xy)$

Question 19. The equation of the tangent plane to the surface $z = xy$ at $x = 1$ and $y = 2$ is

(A) $z = -3 + x + 2y$ (B) $z = x + 2y$ (C) $z = 2$ (D) $z = -4 + 2x + 2y$
(E) $z = -2 + 2x + y$ (F) $z = -1 + x + y$ (G) $z = y$ (H) $z = 1 + x$

Question 20. If $z = x^2y^3$ and $x = -2$, $y = -1$, $dx = .1$, $dy = -.1$, then dz is

(A) 1.6 (B) .016 (C) 2.8 (D) .28d (E) 16
(F) 1.6d (G) 0 (H) 28

PART B

Show all your work in the booklets provided.

Question 21. **$2 \times 5 = 10$**

(a) Find the symmetric equations of the line of intersection of the planes $x + 2y + z = 1$ and $2x - y + z = 2$.

(b) Determine the intersection of the two lines

$$\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-2}{2}, \qquad \frac{x-\frac{1}{4}}{1} = \frac{y+\frac{1}{4}}{3} = z.$$

(c) Find the distance from the point $(1, 0, 1)$ to the plane $2x + y - z = -2$.

(d) Find the plane perpendicular to the plane $x - 2y - z = 1$ and containing the two points $(0, 1, 0)$ and $(1, -2, 1)$.

(e) Find the area of the triangle with vertices $(1, 1, 0)$, $(1, 1, -2)$ and $(0, 2, 0)$.

Question 22. Calculate the curvature, unit tangent, and normal vectors at $t = 0$ for the curve $\vec{r}(t) = (t - \cos t, \sin 2t, t)$. **6**

Question 23. Find all (a, b) such that the tangent plane to $z = x^2 + 2xy + 3y^2$ at (a, b) is normal to the unit vector $(-\frac{2}{3}, -\frac{2}{3}, 1/3)$. **4**